MATH 347 HW 7

due October 26, at the beginning of class

Homework Guildlines

Obviously, your solutions need to be complete and correct, but to receive full credit your write-up should also satisfy the following:

- All the important logical steps in the proof should be present and fully explained.
- All assumptions should be clearly identified.
- Your solutions should be clear and concise. If a sentence does not further the reader's understanding of the solution then it has no place in your write up.
- Use full and grammatically correct English sentences. Mathematical symbols should be used only to render complex mathematical relationships into a readable form.

Moreover, in order to obtain full credit for the homework, you must write down, in the very least, an attempt at a solution for each problem.

Problems

Do the following problems from your book: 4.34, 4.35, 4.47. Additionally, do the following exercise:

(1) Let X be a countably infinite set. Let $\mathscr{P}^{=2}(X)$ denote the following set
$$\mathscr{P}^{=2}(X) := \{A \in \mathscr{P}(X) \mid |A| = 2\}.$$
In other words, $\mathscr{P}^{=2}(X)$ is the collection of subsets of X which have exactly two elements. Show that $\mathscr{P}^{=2}(X)$ is countably infinite.

(2) Let $S \subseteq \mathbb{R}$. Show that a real number α is the least upper bound of S if and only if α is an upper bound for S and if for any positive $\varepsilon > 0$ the number $\alpha - \varepsilon$ is not an upper bound of S