MATH 347 Worksheet 1

Friday 9/14/18

Prove the following.

(1) If $x^2 + y^2$ is even, then $x + y$ is even.
(2) Let a, b, n be natural numbers. Show that if n does not divide ab then n does not divide a and n does not divide b.
(3) Let $x \in \mathbb{Z}$. If $x^2 - 6x + 5$ is even, then x is odd. (there are two proof techniques you can use. Find both proofs.)
(4) Let n be a natural number. Suppose that $n = m\ell$ for natural numbers m and ℓ different from 1. Then m or ℓ is no more than $\sqrt{n}$.
(5) Use the previous problem to show the following: Suppose that n is a composite natural number (i.e. not prime or 1). Then there is a prime divisor of n which is no more than the square root of n.

Also, consider the following.

- (Russell's Paradox) Consider the following expression.

$$U := \{x \mid x \notin x\}.$$

  (1) Discuss what this expression means.
  (2) Show that U cannot possibly be a set. Accomplish this by asking whether or not $U \in U$. What do you find? How does this show that U cannot be a set?

The last problem is known as Russell's paradox. It is meant to indicate why, when we define a set

$$X := \{x \in S \mid P(x)\}$$

we must always specify a set S before hand. To elaborate, we cannot arbitrarily form sets by collecting all sets satisfying $P(x)$, we must confine ourselves to the formation of sets whose elements vary over a *given* set S satisfying some property $P(x)$.