MATH 347 Worksheet 2

Friday 9/21/18

Prove the following.

(1) Suppose that $x_1, \ldots, x_n$ are real numbers in the closed interval $[0, 1]$. Show that we have the inequality

$$\prod_{i=1}^{n}(1 - x_i) \geq 1 - \sum_{i=1}^{n} x_i.$$

(2) Show that for $n \geq 4$ one has $n! > 2^n$.

(3) Show that the identity $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

(4) Let $\{a_i\}_{i=1}^{\infty}$ be a sequence of real numbers. Show that one has the following identity: $|\sum_{i=1}^{n} a_i| \leq \sum_{i=1}^{n} |a_i|$.

(5) Suppose you have a chess board of dimensions $2^n \times 2^n$. Suppose you have L-shaped tiles consisting of three chess squares. Show that you can cover the chessboard with these L-tiles in such a way that you le[11pt]amsart hyperref
[equation]Theorem [equation]Corollary [equation]Lemma [equation]Proposition [equation]Observation [equation]Remark
Theorem Corollary Lemma Proposition Notation
[equation]Definition [equation]Example [equation]Examples [equation]Remark [equation]Claim [equation]Question [equation]Conjecture Exercise
Definition Example Examples Remark Claim Conventions

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(a) Suppose that $x_1, \ldots, x_n$ are real numbers in the closed interval $[0, 1]$. Show that we have the inequality

$$\prod_{i=1}^{n}(1 - x_i) \geq 1 - \sum_{i=1}^{n} x_i.$$

(b) Show that for $n \geq 4$ one has $n! > 2^n$.

(c) Show that the identity $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

(d) Let $\{a_i\}_{i=1}^{\infty}$ be a sequence of real numbers. Show that one has the following identity: $|\sum_{i=1}^{n} a_i| \leq \sum_{i=1}^{n} |a_i|$.

(e) Suppose you have a chess board of dimensions $2^n \times 2^n$. Suppose you have L-shaped tiles consisting of three chess squares. Show that you can cover the chessboard with these L-tiles in such a way that you leave only one specified square uncovered.