MATH 347 Worksheet 3

Friday 9/21/18

Prove the following.

(1) Let $f : X \to Y$ be a function. Suppose that there are functions $g, h : Y \to X$ with the property that $f \circ g = id_Y$ and $h \circ f = id_X$. Show that $g = h$ and that f is hence invertible.

(2) Show that a function $f : X \to Y$ is injective and if and only if there is a function $g : Y \to X$ such that $g \circ f = 1_X$. Likewise, show that f is surjective if and only if there is a function $h : Y \to X$ such that $f \circ h = 1_Y$.

(3) Show that the composition of two injections is injective. Show that the composition of two surjections is surjective. Conclude that the composite of two bijections is a bijection.

(4) Let $a, b, c, d \in \mathbb{R}$ such that $ad - bc \neq 0$ and consider the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by
$$f(x, y) = (ax + by, cx + dy).$$
Show that this function is a bijection.

(5) Show that the sets $\mathbb{N}$ and $\mathbb{N} \setminus \{0\}$ have the same cardinality.

(6) Suppose that $X = A \cup B$ and that we have functions $f : A \to Y$ and $g : B \to Y$. Under what conditions is there a function $h : X \to Y$ with the property that $h(a) = f(a)$ when $a \in A$ and $h(b) = g(b)$ when $b \in B$.

(7) Let $f : X \to Y$ be a function and $A \subseteq X$. The *restriction of f to A* is the unique function $f|_A : A \to Y$ with the property that for each $a \in A$, we have $f|_A(a) = f(a)$. Show that if $f : X \to Y$ is a bijection and if $f(A) = B \subseteq Y$, then $f|_A$ is a bijection from A to B.