MATH 347 Worksheet 3

Friday 9/21/18

Prove the following.

(1) Use the Euclidean algorithm to find the greatest common divisor of the following pairs, and use it to express the gcd as an integer combination
   (a) $(7, 5)$
   (b) $(1071, 462)$
   (c) $(69, 128)$
   (d) $(187, 221)$
(2) Show that if $a|b$ and $b|a$ then $a = b$.
(3) Show that if $a|b$ and $b|c$, then $a|c$.
(4) Show that if $a|b$ and $a|c$, then $a|b + c$.
(5) Here, you will give a nonconstructive proof of the following result (shown in class): Let $a, b \in \mathbb{N}$, then $(a, b) = (d)$ where $d = gcd(a, b)$. Do this in the following steps,
   (a) First, assume that at least one of a, b is nonzero. Define d to be the least positive element in (a, b). Show that $(a, b) = (d)$.
   (b) Show that d is necessarily the greatest common divisor.