MATH 402 Non-Euclidean Geometry
Exam 1 Practice Questions

Other great source for practice questions: the book. More precisely, any exercise or proof from Chapter 1, Chapter 2, Chapter 3, and section 4.1. In addition, review all homework.

1. Definitions and statements
   (a) State Playfair's postulate.
   (b) Define an angle.
   (c) Define congruent triangles.
   (d) Define similar triangles.
   (e) State Pasch's theorem/postulate.
   (f) State the vertical angle theorem.
   (g) State the exterior angle theorem.
   (h) State SAS/SSS/ASA congruence rule.
   (i) Define orthogonal circles.
   (j) Define the inverse of a point with respect to a given circle.
   (k) Define a tangent line to Γ.
   (l) Define when two circles are tangent to each other.
   (m) Define the interior of an angle.
   (n) Define the interior of a circle.
2. Euclidean constructions: Provide constructions of the following
   (a) Equilateral triangle with a given side.
   (b) Angle bisector.
   (c) Given a circle Γ and two points A, B, another circle Δ which is orthogonal to Γ and passes through the given points A, B.
   (d) Given a circle Γ and a point P on Γ, construct a tangent to Γ through P.

3. Proofs

   (a) Prove the vertical angle theorem.

   (b) Prove the exterior angle theorem.

   (c) Prove that Playfair's postulate implies the following statement: If l_1 and l_2 are two unequal parallel lines, and m is another line which intersects l_1 (but is not equal to l_1), then m also intersects l_2.

   (d) Given Hilbert's axioms, prove SSS.

   (e) Consider the axiomatic system defined by the following. The undefined terms are points, and a line is defined as a set of points. The axioms are:

      i. There are exactly four points.

      ii. There are exactly four lines.

      iii. Given any two different points there exists at least one line that contains them.

      - I claim that this system is consistent. Give a model.
      - Show that each of these three axioms is independent from the others.
      - Is the system complete? Why or why not?
      - Is the following statement true or false for the system: Every line contains at most two points. Justify your answer.
      - From the given system S form a new one T which is "dual," i.e. points in T are lines in S, and lines in T are points in S. Does T satisfy the same axioms as S? If the answer is yes, prove your claim by proving each axiom holds for T. If the answer is no, prove that one of the axioms for S (which one?) does not hold.