MATH 402 Non-Euclidean Geometry

Exam 2 Practice Questions

Other great source for practice questions: the book. More precisely, any exercise or proof from Chapter 1, Chapter 2, Chapter 3, and section 4.1. In addition, review all homework.

1. Definitions and statements
   (a) Definition of an isometry.
   (b) Define what a fixed point is.
   (c) Define what an invariant line is.
   (d) Define a reflection.
   (e) Define a translation.
   (f) Define a rotation.
   (g) Define a glide reflection
   (h) Define the addition of two vectors.
   (i) Define sine and cosine.
   (j) State the law of sines.
   (k) State the law of cosines.
   (l) Define the dot product.
   (m) State the correspondence between the dot product and angles.

2. Proofs

   Let l, m, n be three lines, and r_l, r_m, r_n the associated reflections.

   (a) If the intersection $l \cap m \cap n$ is a point O, what kind of isometry is $r_n \circ r_m \circ r_l$? Prove your answer.

   (b) If l, m, n are all parallel to each other, what kind of isometry is $r_n \circ r_m \circ r_l$? Prove your answer.

   (c) Let f be any isometry, and r_l reflection about a line l. Show that the composition $f \circ r_l \circ f^{-1}$ is reflection about the line $f(l)$.

(d) Let f be any isometry, and $T_{\vec{v}}$ translation by vector $\vec{v}$. Show that the composition $f \circ T_{\vec{v}} \circ f^{-1}$ is translation by vector $f(\vec{v})$.

(e) Suppose l and m are lines that intersect at a point O. What is the angle of rotation for the composition $r_m \circ r_l$? Prove your answer.

(f) Show that the composite of rotations is again a rotation.

(g) Show that the rotations form a subgroup of the isometries.