MATH 402 Worksheet 2

Friday 2/2/18

Exercise 1. Show that Playfair's postulate is equivalent to Euclid's fifth postulate.

Exercise 2. In Euclid's elements, the concept of *sidedness* is used quite frequently. In this exercise you will make this concept mathematically rigorous within Hilbert's axiomatic system.

(1) List out the undefined terms of Hilbert's axioms.
(2) Write down the axioms of incidence, are they consistent?
(3) Write down the axioms of betweeness. (Denote the relation B is between A and C by $A * B * C$).
(4) Recall the definition of an equivalence relation and of an equivalence class.
(5) Can you figure out why I asked you to recall the definition of an equivalence relation? Use this to come up with a definition of sidedness.
(6) Prove that your definition gives an equivalence relation.
(7) How many equivalence classes ought there be? Prove that your intuition gives you the right number.

Exercise 3. Using the previous exercise, you will prove the *Crossbar Theorem*: Let $\angle ABC$ be an angle, and let D be a point in the *interior* of the angle. Show that the ray $\overrightarrow{AD}$ intersects the segment $\overline{AB}$.

(1) Draw a picture illustrating the content of this theorem.
(2) Define what an angle is, and define what ought to be meant by the interior.
(3) Provide a proof of the crossbar theorem. It may be helpful to try to reason with pictures first, and then write down a rigorous proof.

You do not need to turn in this worksheet. However, you should make sure to work out the problems. The problems that appear on worksheets may appear on the examines, so be sure to work out solutions even if you don't finish in class. Come to office hours if you have questions.