MATH 402 Worksheet 4

Friday 2/16/18

Exercise 1. Suppose that Γ and Γ' are circles with center O and O' respectively. We say that Γ and Γ' are *mutually tangent* at a point T if they share a common tangent line passing through T. Suppose that Γ and Γ' are mutually tangent circles which lie on opposite sides of the common tangent (we say they are *externally tangent at* T in this case), show that the line OO' passes through T. (Hint: You may assume the *triangle inequality*.)

Exercise 2. Given a line ℓ and a point B on ℓ and a point A not on ℓ, construct a circle passing through A and B which is tangent to the line ℓ.

Exercise 3. Using Hilbert's axioms (specifically the ones for congruence of line segments), show the following:

(1) Given three points A, B, C on a line with $A * B * C$, and given points E, F on a ray originating from D, suppose that $DE \cong AB$ and $AC \cong DF$. Then E will be between D and F and $BC \cong EF$. For this reason, we regard BC as the difference of AC and AB.

(2) Provide a definition for the sum of line segments: Given segments AB and CD, define what one means by $AB + CD$.

(3) Try to prove the following: Given $AB \cong A'B'$ and $CD \cong C'D'$, then $AB + CD \cong A'B' + C'D'$.