

# MATH 402 Worksheet 8

Friday 4/13/18

**Definition 1.** A *Saccheri quadrilateral*  $ABCD$  with congruent sides  $AC$  and  $BD$ , such that both sides are perpendicular to the base  $AB$ . The *midline* is the line segment joining the midpoint of  $AB$  to the midpoint of  $CD$ .

**Exercise 1.** Show that the top angles of a Saccheri quadrilateral are congruent and that the midline intersects  $AB$  and  $CD$  at right angles.

**Exercise 2.** Let  $ABCD$  be a quadrilateral so that the sides  $AC$  and  $BD$  intersect  $AB$  at right angles, but so that the sides are not congruent. Show that the angle at  $C$  is greater than the angle at  $D$  if and only if  $AC < BD$ .

**Exercise 3.** Let  $ABCD$  be a Saccheri quadrilateral. Let  $P$  be a point on the segment  $CD$  and let  $PQ$  be perpendicular to  $AB$ . Let  $\alpha$  be the angle at  $A$  (which is equal to the angle at  $D$ ). Show that

- (1) If  $PQ < BD$  then  $\alpha$  is acute,
- (2) If  $PQ = BD$ , then  $\alpha$  is a right angle, and
- (3) If  $PQ > BD$ , then  $\alpha$  is obtuse.