

MATH 402 Worksheet 2

Friday 2/2/18

Exercise 1. Hilbert includes in his system the following incidence.

- (1) for any two distinct points, there is a unique line ℓ containing A and B ,
- (2) Every line contains at least two points.
- (3) There are three non-collinear points.

Show that two lines ℓ and m intersect at at most one point. Additionally,

Exercise 2. In Euclid's elements, the concept of *sidedness* is used quite frequently. In this exercise you will make this concept mathematically rigorous within Hilbert's axiomatic system.

- (1) List out the undefined terms of Hilbert's axioms.
- (2) Write down the axioms of betweenness. (Denote the relation B is between A and C by $A * B * C$).
- (3) Recall the definition of an equivalence relation and of an equivalence class.
- (4) Can you figure out why I asked you to recall the definition of an equivalence relation? Use this to come up with a definition of sidedness.
- (5) Prove that your definition gives an equivalence relation.
- (6) How many equivalence classes ought there be? Prove that your intuition gives you the right number.

Exercise 3. Using the previous exercise, you will prove the *Crossbar Theorem*: Let $\angle ABC$ be an angle, and let D be a point in the *interior* of the angle. Show that the ray \overrightarrow{AD} intersects the segment \overline{AB} .

- (1) Draw a picture illustrating the content of this theorem.
- (2) Define what an angle is, and define what ought to be meant by the interior.
- (3) Provide a proof of the crossbar theorem. It may be helpful to try to reason with pictures first, and then write down a rigorous proof.

You do not need to turn in this worksheet. However, you should make sure to work out the problems. The problems that appear on worksheets may appear on the examines, so be sure to work out solutions even if you don't finish in class. Come to office hours if you have questions.