MATH 402 Worksheet 3

Friday 2/9/18

Exercise 1.

(1) Discuss and write down Hilbert's axioms for the congruence of line segments and angles.
(2) Show that congruence defines an equivalence relation.
(3) Given points A, B, C on a line, provide a definition for $AB < AC$. Extend this to a definition of of $AB < CD$ for line segments AB and CD.

Exercise 2. Show that the sum of interior angles of a triangle is always two right angles.

Exercise 3. Suppose that Γ is a circle with center O and radius OA, and Γ' is a circle with center O' and radius $O'A'$. Show that if the circles Γ and Γ' coincide on the level of points, then $O = O'$. In other words, the center of a circle is uniquely determined. (Hint: You will need to make use of one of Hilbert's axioms.)

Exercise 4. Recall that the Cartesian plane is $\mathbb{R}^2$, which is the set of ordered pairs of real numbers,

$$\mathbb{R}^2 = \{(a, b) \mid a \in \mathbb{R},\, b \in \mathbb{R}\}.$$

In this exercise, you will check that several of Hilbert's axioms hold in the Cartesian plane.

(1) Provide an interpretation for the notions of point, line, and circle in the Cartesian plane.
(2) Given your notion of point, show that the axioms (I1)-(I3) and Playfair's axiom hold in $\mathbb{R}^2$.
(3) Provide an interpretation of betweeness in the Cartesian plane. Show that the axioms (B1)-(B4) hold in the Cartesian plane.