MATH 402 Worksheet 9

Friday 4/13/18

Exercise 1. Show that in a hyperbolic plane, the summit angles of a Saccheri quadrilateral are acute. Use this to show the sum of the interior angles in a hyperbolic plane is less than two right angles.

Definition 1. Define the defect of a triangle ABC to be

$$\delta(ABC) := 2RA - (\text{sum of the interior angles of } ABC)$$

For a quadrilateral $ABCD$ define the defect to be

$$\delta(ABCD) := 4RA - (\text{sum of the interior angles of } ABCD)$$

Exercise 2. Show that δ has some additive properties. Namely, if $ABCD$, then the defect of $ABCD$ is the sum of the defect of ABC and BCD. Also, show that if ABC is a triangle with D a point on AB and E a point on AC, then the defect of ABC is the defect of ADE plus the defect of $DEBC$.

Exercise 3. Show that in the hyperbolic plane that if two triangles have corresponding angles congruent, then they are themselves congruent. (Hint: Use the previous exercises).