MATH 402 Worksheet 7

Friday 4/6/18

Exercise 1. Recall two circles are called orthogonal circles if their tangent lines at their points of intersection are orthogonal to each other. Recall that a point A not on a circle Γ (with center O) has an *inverse*. This is the point A' on the ray $\overrightarrow{OA}$ such that $OA \cdot OA' = r^2$, where r is the radius of Γ. Since the relation is symmetric in A and A', note that A is the inverse of A'.

(1) Let A be a point in the interior of the circle Γ. Let PQ be the chord which intersects the line $\overleftrightarrow{OA}$ at A at right angles. Let ℓ and m be the tangent lines to Γ passing through P and Q respectively. Let A' be the point of intersection. Show that A and A' are inverses with respect to Γ. (Hint: Use similar triangles.)

(2) Let A and A' be points which are inverse with respect to Γ. Suppose Δ is a circle which passes through A and A'. Show that Δ and Γ intersect at right angles.

(3) Show that, given any two points P And Q in the interior of a circle Γ, there is a unique circle Δ which passes through P and Q which is orthogonal to Γ.

(4) Explain how this shows that in the Poincaré disc, any two points are connected by a unique hyperbolic line.

Exercise 2.

(1) Consider the Poincaré disc. Let ℓ be a *hyperbolic line*, and let P be a point not on the line. Show that there is a unique hyperbolic line m passing through P which intersects ℓ at the boundary.

(2) Consider the line you constructed in the last part. Show that if Q is a point on ℓ and R is some point on m which is different from P, then there is no hyperbolic line k in the interior of $\angle APQ$ which is parallel to ℓ.